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## GEOMETRY.

**426. Proposed by R. D. CARMICHAEL, Indiana University.**

On a given chord of a circle as a base construct an isosceles triangle, with vertex outside of the circle, such that its sides shall be divided in a given ratio by their points of intersection with the circle.

## CALCULUS.

**347. Proposed by R. D. CARMICHAEL, Indiana University.**

Show that the differential equation

$$9\left(\frac{d^2y}{dx^2}\right)^2\frac{d^5y}{dx^5}-45\frac{d^2y}{dx^2}\frac{d^3y}{dx^3}\frac{d^4y}{dx^4}+40\left(\frac{d^3y}{dx^3}\right)^3=0$$

remains unchanged when the variables $x$ and $y$ undergo any projective transformation (Goursat-Hedrick, *Mathematical Analysis,* p. 86, ex. 18).

**348. Proposed by E. L. DODD, University of Texas.**

Let $(x_1, x_2, \cdots x_n)$ be a point in $n$ dimensions lying in the "sphere" $S$ defined by

$$x_1^2+x_2^2+\cdots+x_n^2 \leqq 1.$$

Let $T$ be that part of $S$ defined by a set of $n$ linear homogeneous inequalities with non-vanishing determinant; thus:

$$a_ix_1+b_ix_2+\cdots+k_ix_n \geqq 0, \quad i=1, 2, \cdots n.$$

Find the value of

$$\frac{\int \underset{T}{\cdots} \int dx_1 \cdots dx_n}{\int \underset{S}{\cdots} \int dx_1 \cdots dx_n};$$

in other words, find the magnitude of a "solid angle" in $n$ dimensions, with the "sphere" as unit solid angle.

*Note.*—This problem was discussed and left unsolved by Schläfli in the Quarterly Journal of Mathematics for 1858, 1860, 1867. EDITOR.

**349. Proposed by C. N. SCHMALL, New York City.**

If $y=a\cos(\log x)+b\sin(\log x)$, eliminate the constants $a$ and $b$ and obtain the equation

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0.$$

## MECHANICS.

**282. Proposed by R. P. LOCHNER, Philadelphia, Pa.**

A car weighing 10 tons (2,240 lb. each) attains a speed of 15 miles per hour from rest in 24 seconds, during which it covers 100 yards. If the space-average of the resistances is 30 lb. per ton, find the average horse-power used to drive the car. (Morley's "*Mechanics for Engineers,*" p. 66).

**283. Proposed by C. N. SCHMALL, New York, N. Y.**

The maximum length of a certain chain which can be suspended from one end without breaking is $l$. It is desired to form a catenary with a length $2l/k$ of the chain, the points of support being a distance $d$ apart, in the same horizontal line.

Show that the maximum value of $d$ is $\frac{2l}{k}(k^2-1)^{\frac{1}{2}}\log\left(\frac{k+1}{k-1}\right)^{\frac{1}{2}}$.

## NUMBER THEORY AND DIOPHANTINE ANALYSIS.

**199. Proposed by R. P. LOCHNER, Philadelphia, Pa.**

Find three integral squares such that the sum of every two of them shall be a square.—Alsop's "Algebra."

**200. Proposed by R. D. CARMICHAEL, Indiana University.**

Find the general solution, in relatively prime integers, of the equation $x^2 + y^2 = z^4$.

**201. Proposed by E. T. BELL, Seattle, Washington.**

Eisenstein (*Crelle*, t. 27, p. 282) proposed, as the simplest of several problems: "In the expansion of

$$\frac{1 + z + z^2 + \cdots + z^{p-1}}{(1 - z)^{p-1}} - 1,$$

where $p$ is prime, to show that the coefficients of the various powers of $z$ are all divisible by $p$."

**202. Proposed by A. R. SCHWEITZER, Chicago, Ill.**

There exists an infinitude of systems of dyads $\{\alpha\beta\}$ in 7, 9, 11, etc., elements such that each system has the following properties: (1) if $\alpha\beta$ is in the set, then $\beta\alpha$ is not in the set; (2) for each dyad $\alpha\beta$ in the set there exists an element $\xi$ such that $\xi\beta$ and $\alpha\xi$ are also in the set. For example, such a system is,

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 12, | 23, | 34, | 45, | 56, | 67, | 78, | 89, | 91 |
| 13, | 24, | 35, | 46, | 57, | 68, | 79, | 81, | 92 |
| 14, | 25, | 36, | 47, | 58, | 69, | 71, | 82, | 93 |
| 51, | 62, | 73, | 84, | 95, | 16, | 27, | 38, | 49 |

Investigate the existence of

I. A finite set of triads $\{\alpha\beta\gamma\}$ such that (1) if $\alpha\beta\gamma$ is in the set, then $\beta\gamma\alpha$, $\gamma\alpha\beta$ are also in the set but $\beta\alpha\gamma$ is not in the set, (2) for each triad $\alpha\beta\gamma$ in the set there exists an element $\xi$ such that $\xi\beta\gamma$, $\alpha\xi\gamma$, $\alpha\beta\xi$ are also in the set.

II. A finite set of tetrads $\{\alpha\beta\gamma\delta\}$ such that (1) if $\alpha\beta\gamma\delta$ is in the set, then $\beta\gamma\alpha\delta$, $\gamma\alpha\beta\delta$, $\gamma\delta\alpha\beta$ are also in the set but $\beta\alpha\gamma\delta$ is not in the set, (2) for each tetrad $\alpha\beta\gamma\delta$ in the set there exists an element $\xi$ such that $\xi\beta\gamma\delta$, $\alpha\xi\gamma\delta$, $\alpha\beta\xi\delta$, $\alpha\beta\gamma\xi$ are also in the set.

The problem for alternating *n-ads* for $n > 4$ is obvious.

## SOLUTIONS OF PROBLEMS.

### ALGEBRA.

**382. Proposed by C. E. FLANAGAN, Wheeling, W. Va.**

A few days ago, I deduced the following formula for finding the value of the unknown quantity in a cubic equation having the form, $x^3 + 3A^2x = B$.

Let $C = \sqrt{\frac{2B}{A^3} + 1} - 1$. Then $x = \frac{2B}{A^2(C^2 + 12)} + \frac{AC}{4}$.

It is required to show: (1) How this formula was derived; (2) under what conditions does it give exact results; (3) in general, what is its degree of approximation; (4) if possible, modify it so that it will always give exact results; (5) if it cannot be so modified, show why.

### Solution by the Proposer.

(1) The formula was derived by equating the volume of a spherical segment of one base and unknown altitude, $x$, to $n$ times the volume of a sphere whose radius is the radius of the base of the spherical segment.

Thus, let $r$ be the given radius of the base of the spherical segment of one base and $x$ the unknown altitude. Then we have by hypothesis

$$\tfrac{1}{3}\pi x^2(3r - x) = n(\tfrac{4}{3}\pi r^3) = \tfrac{1}{2}\pi x r^2 + \tfrac{1}{6}\pi x^3.$$

Hence,

$$3rx^2 - x^3 = 4nr^3,$$

and

$$3r^2x + x^3 = 8nr^3;$$

whence, by adding the two equations and reducing,

$$r^2x + rx^2 = 4nr^3.$$